EVALUATION OF SUPER-TSD NETWORK-ANALYZER CALIBRATION PROGRAMS BY COMPUTER SIMULATION

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ABSTRACT

Two Fortran programs, which implement the SUPER-TSD calibration method for automated network analyzers, have been written for the conventional case of two-port scattering parameter measurements. These SUPER-TSD error-computation and error-removal programs are being evaluated by using computer-generated simulated calibration data representing known system errors, including leakage and switching errors. The results obtained so far confirm all the predictions of the SUPER-TSD theory, including the theoretically unlimited capability of removing leakage errors. This capability is limited in real measurements only by the finite system resolution and stability and by noise.

Introduction

The recently introduced SUPER-TSD method[1,2] for the calibration of automated network analyzers differs from the original TSD method[3,4] in two respects:

1. The SUPER-TSD error model accounts for possible leakage errors.
2. The SUPER-TSD error model is generalized to cover N-port scattering parameter measurements.

Two Fortran programs, which implement this new calibration method, have already been written for the fundamental case of two-pórt scattering parameter measurements.

In this particular case, N = 2 and the error model reduces to a virtual four-port error network E_N having ports 1 and 2 connected to the ANA and ports 3 and 4 connected to the device under test X (Figs. 1a/1b).

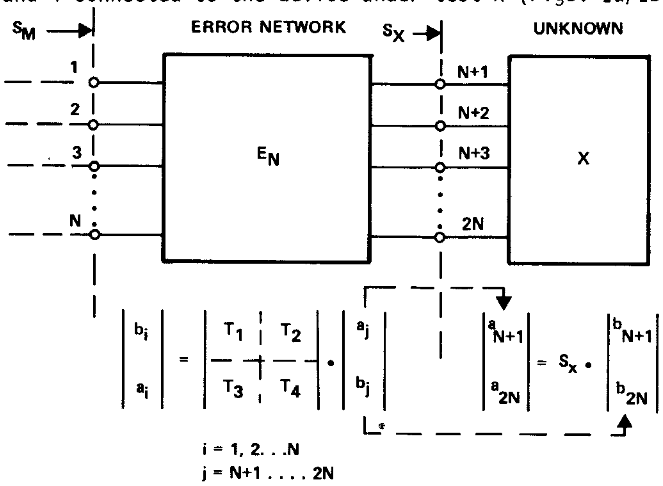

Figure 1a. The SUPER-TSD error model is a virtual 2N-port error network assumed to be connected between the unknown n-port X and an ideal n-port ANA.

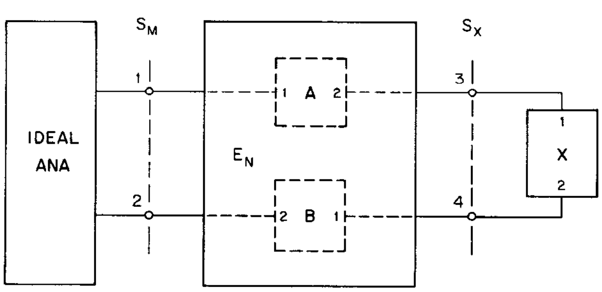

Figure 1b. Reduced SUPER-TSD error model for two-port measurements: Dashed, inside the four-port E_N is the zero-leakage TSD error model consisting of the "error two-ports" A and B.

In the absence of leakage errors, the four-port error network E_N reduces to the TSD error model consisting of two error two-ports A and B (Fig. 1b).

The first SUPER-TSD Fortran program accepts as input data the measured scattering parameters of three two-port standards and computes the scattering response of the virtual four-port error network in terms of any or all of three 4x4 complex matrices.[2]

These matrices are:

1. The T-parameter matrix T.
2. The inverse $R = T^{-1}$ of the T-parameter matrix T.
3. The scattering matrix S.

The two-port standards assumed to be used are the same as for the original TSD method: a "Through," a "Short," and a "Delay." The "Through" and the "Delay" are known lengths of lossless transmission line with nominal impedance replacing the unknown X. The "short" is a pair of perfect shorts connected at ports 3 and 4.

The second SUPER-TSD Fortran program accepts as input data the inverse R of the T-matrix of the virtual error network E_N and the uncorrected scattering parameters S_M of a device under test as measured by the ANA. It then computes the corrected scattering parameters S_X of this device by deembedding the error-network response.

These two SUPER-TSD programs are being tested and evaluated by computer simulation. The simulation is performed by assuming the virtual error network to have a known equivalent circuit, including leakage paths between ports 1 and 2, 3 and 4, 1 and 4, and 2 and 3. The measured scattering parameter matrices S_{Mi} of the three two-port standards and S_M of a known two-port network, representing the device under test, are then computed using a network analysis program. These scattering parameter matrices are then used as simulated calibration data and simulated uncalibrated measurements for the SUPER-TSD error-computation and error-removal programs, respectively.

The evaluation of the two programs is then performed by a quantitative comparison between the corrected scattering parameters of the device under test as obtained from these programs and the theoretical scattering parameters of the known two-port network assumed to represent it. This theoretical response is computed from the network configuration and parameter values.

2. The SUPER-TSD Error-computation Program

The explicit matricial solutions for the T-parameter matrix T and its inverse $R = T^{-1}$, implemented in the SUPER-TSD error-computation program, are expressed in terms of the "Reshuffle" RS(..) and the "Stacking" S(..) operators, respectively, of their four quadrants:

$$RS(T_4) = \text{Complex Column Vector of order } N^2 \ (N = 2) \quad (1)$$

$$RS(T_1) = P_1 \cdot RS(T_4) = \left[(B_2A_2^{-1} - D_2C_2^{-1})(E_2A_2^{-1} - F_2C_2^{-1})^{-1}\right]^T \cdot RS(T_4) \quad (2)$$

$$RS(T_2) = P_2 \cdot RS(T_4) = \left[(A_3B_2^{-1} - C_3D_2^{-1})(E_3B_2^{-1} - F_3D_2^{-1})^{-1}\right]^T \cdot RS(T_4) \quad (3)$$

$$RS(T_3) = P_3 \cdot RS(T_4) = -\left[(B_2E_2^{-1} - D_2F_2^{-1})(A_2E_2^{-1} - C_2F_2^{-1})^{-1}\right]^T \cdot RS(T_4) \quad (4)$$

$$S(R_1) = \text{Complex Column Vector of Order } N^2 \ (N = 2) \quad (5)$$

$$S(R_2) = Q_1 \cdot S(R_1) = -(B_2^{-1}E_3 - D_2^{-1}F_3)^{-1} \cdot (B_2^{-1}A_3 - D_2^{-1}C_3) \cdot S(R_1) \quad (6)$$

$$S(R_3) = Q_2 \cdot S(R_1) = (E_2^{-1}A_2 - F_2^{-1}C_2)^{-1} \cdot (E_2^{-1}B_2 - F_2^{-1}D_2) \cdot S(R_1) \quad (7)$$

$$S(R_4) = Q_3 \cdot S(R_1) = (A_2^{-1}E_2 - C_2^{-1}F_2)^{-1} \cdot (A_2^{-1}B_2 - C_2^{-1}D_2) \cdot S(R_1) \quad (8)$$

where the ten $N^2 \times N^2$ auxiliary matrices $A_2, B_2 \ldots F_2, A_3 \ldots F_3$ are expressed by differences of Kronecker tensor products:

$$A_2 = (S_{M1}^T \otimes S_{S1}) - (S_{M2}^T \otimes S_{S2}) \quad (9)$$

$$B_2 = (S_{M1}^T \otimes I) - (S_{M2}^T \otimes I) \quad (10)$$

$$C_2 = (S_{M1}^T \otimes S_{S1}) - (S_{M3}^T \otimes S_{S3}) \quad (11)$$

$$D_2 = (S_{M1}^T \otimes I) - (S_{M3}^T \otimes I) \quad (12)$$

$$E_2 = (I \otimes S_{S1}) - (I \otimes S_{S2}) \quad (13)$$

$$F_2 = (I \otimes S_{S1}) - (I \otimes S_{S3}) \quad (14)$$

$$A_3 = (S_{M1}^T \otimes S_{S1}^{-1}) - (S_{M2}^T \otimes S_{S2}^{-1}) \quad (15)$$

$$C_3 = (S_{M1}^T \otimes S_{S1}^{-1}) - (S_{M3}^T \otimes S_{S3}^{-1}) \quad (16)$$

$$E_3 = (I \otimes S_{S1}^{-1}) - (I \otimes S_{S2}^{-1}) \quad (17)$$

$$F_3 = (I \otimes S_{S1}^{-1}) - (I \otimes S_{S3}^{-1}) \quad (18)$$

Here I is the unit matrix and the superscript T indicates matrix transposition.

The S_{Mi} and S_{Si} N x N matrices appearing in the Kronecker products are the measured and the postulated scattering matrices of the three two-port calibration standards, respectively. In the program, standard #1 is a "Through"; #2, a "Short"; and #3, a "Delay."[2]

Mathematical proof has been found of the arbitrariness of the T_4 and R_1 matrix quadrants, with the only restriction being nonsingularity. This arbitrariness has also been confirmed numerically by running the error-computation program with alternative definitions of these quadrants. These lead to mutually equivalent T and R matrices, all of which transform a given object S-matrix S_X into the same measured S-matrix S_M (for T) or a given S_M into the same S_X (for R).

The S-matrix of the error network is computed only upon operator's choice, for the sake of visualizing the system's errors in a familiar form of representation. The S-matrix may be computed from either the T-matrix or the R-matrix using the expressions:

$$S = \left|\begin{array}{c|c} T_2T_4^{-1} & T_1 - T_2T_4^{-1}T_3 \\ \hline T_4^{-1} & -T_4^{-1}T_3 \end{array}\right| \quad (19)$$

$$S = \left|\begin{array}{c|c} -R_1^{-1}R_2 & R_1^{-1} \\ \hline R_4 - R_3R_1^{-1}R_2 & R_3R_1^{-1} \end{array}\right| \quad (20)$$

3. The SUPER-TSD Error-removal Program

The deconvolution of the error-network response from the measured scattering matrix S_M is performed through a fractional bilinear transformation in matrix form using the quadrants of the R-matrix:

$$S_X = (R_1 \cdot S_M + R_2)(R_3 \cdot S_M + R_4)^{-1} \quad (21)$$

The SUPER-TSD error-removal program actually may, upon operator's choice, perform the corresponding inverse transformation:

$$S_M = (T_1 \cdot S_X + T_2)(T_3 \cdot S_X + T_4)^{-1} \quad (22)$$

This is easily accomplished due to the formal identity of (21) and (22).

The transformation (21) is referred to as "deembedment" as opposed to (22), which is called "embedment."

4. Results of Program Evaluations

Two different networks have been used as the postulated device under test in the evaluation of the SUPER-TSD programs:

1. A nonsymmetric resistive "T" pad designed to have $S_{11} = -0.5$, $S_{22} = 0.25$, and $S_{12} = S_{21} = 0.125$ (-6 dB, -12 dB, and -18 dB, respectively).
2. A lumped-element, five-resonator Tchebicheff filter with a 0.5-dB ripple from 2 to 4 GHz and a steep response skirt on either side.

A first test was run upon a situation that had already been used in a similar evaluation of TSD programs. In this situation, the four-port error network E_N reduces to two error two-ports A and B, and there is no leakage (Fig. 1b). The assumed error two-ports are 100-ohm quarter-wave transformers with short, 50-ohm and 200-ohm lines on either side.[5]

The calibration was assumed to be performed using a 20-ps, 200-ohm line for the "Through" and a 100-ps 200-ohm line for the "Delay," plus perfect "Shorts." The device under test was a 6-12-18 dB nonsymmetric "T" pad designed for 200-ohm impedance level. This zero leakage test proved two facts:

1. All quadrants of the T and R matrices are, in this case, diagonal as theoretically predicted. Rounding off errors prevented the off-diagonal entries from being flat zero. These entries, however, were below -150 dB.
2. The T and R matrices obtained are both normalized to 200 ohms at ports 3 and 4, the calibration interfaces, as a consequence of using 200-ohm lines in the "Through" and "Delay" calibration measurements.

This means that, in this case, after deembedment the obtained S_X scattering matrix of the device under test is normalized to 200 ohms, the impedance of the standards used. By recomputing the resistance values of the pad from the reconstructed S_X matrix, deviations were observed with respect to the originally assumed values. These are in the order of one part in 10^6.

Three different tests were run by assuming the four-port error network to be represented by a 3-dB hybrid connected in either of two different ways between the ideal ANA and the device under test. A 50-ohm 6-12-18 dB resistive pad and a Tchebicheff filter were assumed as device under test.

Depending upon how the hybrid is connected, eight zero entries appear in different locations of the T and R matrices, but four non-zero leakage entries are always present.

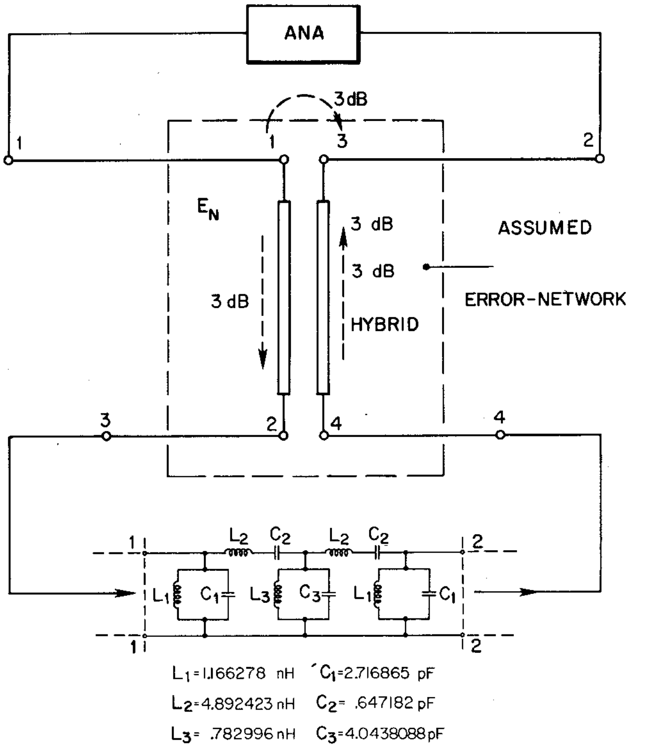

Figure 2. Simulated measurement of a bandpass filter against massive leakage errors.

The most representative result has been obtained from the situation shown in Fig. 2 Here the five-resonator Tchebicheff filter is assumed to be measured through the 3-dB hybrid, while this introduces a leakage signal of -3 dB that massively overshadows the filter skirt response.

Figs. 3 and 4 illustrate the almost incredible result obtained. The curve labeled R (raw data) in Fig. 3 is the response of the filter as seen from the ANA through the 3-dB hybrid. The curve labeled C (corrected data) shows the response of the filter as reconstructed through deembedment down to a skirt response of -170 dB. Fig. 4 shows how this reconstructed response is for all practical purposes indistinguishable from the theoretical response of the filter (second curve, dashed, and labeled T). This means a leakage error, occasionally 150 dB above the device response, has been accurately removed.

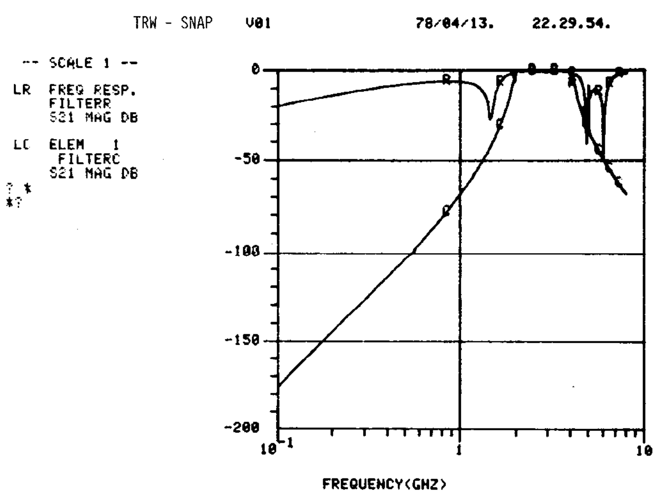

Figure 3. Results of the simulated leakage situation, shown in Fig. 2: R = raw data, C = corrected filter response.

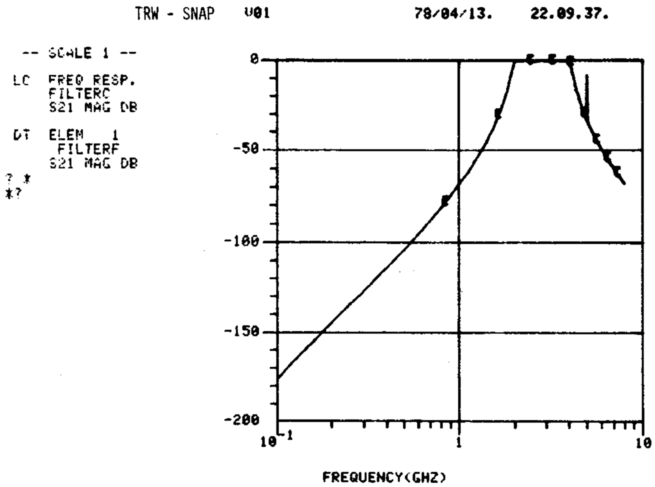

Figure 4. The corrected filter response is indistinguishable from its theoretical response: C = corrected data, T = theoretical response.

It becomes then worthwhile to redesign existing ANA hardware for largely improved stability and resolution, as these will be the only factors limiting this capability in practice.

Conclusion

Results obtained so far prove a fast and correct execution of both programs without incurring any matrix singularity problems at any point of the mathematical process. Typical running time is 20 seconds for 200 frequency points on a CDC 6600. In particular, the theoretically unlimited capability of removing leakage errors has been numerically confirmed. Full details of this evaluation will be given in the transaction paper.

References

[1] N. R. Franzen and R. A. Speciale, "Super-TSD. A Generalization of the TSD Network Analyzer Calibration Procedure, Covering N-port Measurements with Leakage," 1977 IEEE-MTT-S International Microwave Symposium, San Diego, California, Digest of Technical Papers, IEEE Cat. No. 77CH1219-5, pp. 114-117, June 21-23, 1977.

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[3] N. R. Franzen and R. A. Speciale, "A New Procedure for System Calibration and Error Removal in Automated S-parameter Measurements," Proceedings of the 5th European Microwave Conference, Hamburg, Germany, 1-4 September 1975, pp. 69-73, Publ.: Microwave Exhibitions and Publishers, Sevenoaks, Kent, England.

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[5] R. A. Speciale, "Automated Component Testing--It's a Reality...Thanks to Minicomputers," Microwave Syst. News, Vol. 6, pp. 79-99, June/July 1976.